By the interchangeability of partial derivatives,
\[ \frac{\partial}{\partial v} \left( \frac{\partial u}{\partial s} \right) = \frac{\partial}{\partial s} \left( \frac{\partial u}{\partial v} \right) \]

The thermodynamic identity for \( U \) (eqn 5.16) says
\[ dU = T \, ds - P \, dV + \mu \, dN \]

So if \( V, N \) are held fixed, \( \left( \frac{\partial u}{\partial s} \right)_{V,N} = T \)

And if \( S, N \) are held fixed, \( \left( \frac{\partial u}{\partial v} \right)_{S,N} = -P \)

Plugging these into the partial derivative formula, we have
\[ \frac{\partial}{\partial v} (T)_{S} = \frac{\partial}{\partial s} (-P)_{V} \]

\[ \Rightarrow \left( \frac{\partial T}{\partial v} \right)_{S} = -\left( \frac{\partial P}{\partial s} \right)_{V} \]

Now we are supposed to do something similar for the other "energies". So what was special about the starting point? Well by the thermodynamic identity for \( U \), we are urged to consider \( U(S,V,N) \). Taking \( N \) out of the picture, we see the cross densities are those of the remaining variables.

So let's move on to enthalpy:
\[ H (S, P, N) = \text{try} \]

\[ \frac{\partial}{\partial S} \left( \frac{\partial H}{\partial P} \right) = \frac{\partial}{\partial P} \left( \frac{\partial H}{\partial S} \right) \]

From the thermodynamic identity for \( H \) (Eq. 5.18), we see:

\[ (\frac{\partial H}{\partial P}) = V, \quad (\frac{\partial H}{\partial S}) = T. \]

Thus, \( (\frac{\partial V}{\partial S})_P = (\frac{\partial T}{\partial P})_S \) in the resulting Maxwell relation.

Now for the Helmholtz free energy \( F (T, V, N) \)

\[ \frac{\partial}{\partial T} \left( \frac{\partial F}{\partial V} \right) = \frac{\partial}{\partial V} \left( \frac{\partial F}{\partial T} \right) \]

By the thermodynamic identity for \( F \), we see

\[ \frac{\partial F}{\partial V} = -P, \quad \frac{\partial F}{\partial T} = -S \]

\[ \Rightarrow \quad (\frac{\partial P}{\partial T})_V = (\frac{\partial S}{\partial V})_T \]

Finally, the Gibbs free energy \( G (T, P, N) \)

\[ \frac{\partial}{\partial T} \left( \frac{\partial G}{\partial P} \right) = \frac{\partial}{\partial P} \left( \frac{\partial G}{\partial T} \right) \]

but \( (\frac{\partial G}{\partial P}) = V, \quad (\frac{\partial G}{\partial T}) = -S \)
$$S_0 \left( \frac{\partial v}{\partial T} \right)_p = -\left( \frac{\partial S}{\partial p} \right)_T$$