2.31 Start with equation 2.40

\[ \Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \frac{\pi^{3N/2}}{(3N/2)!} \left( \frac{2\mu U}{h^3} \right)^{3N/2} \]

Now apply Stirling's approximation for the factorials.

\[ \Omega_N = \frac{1}{N^N e^{-N} \sqrt{2\pi N}} \frac{V^N}{h^{3N}} \frac{\pi^{3N/2}}{(3N/2)^{3N/2}} e^{-3N/2} \frac{\left(\frac{2\mu U}{h^3}\right)^{3N/2}}{\sqrt{2\pi (3N/2)}} \]

Knowing where we are going, rewrite as powers of \( N \)

\[ \Omega_N = \frac{e^N V^N}{N^N} \frac{\left( \frac{\pi^{3/2}}{\sqrt{2\pi N}} \right)^N}{\left( \frac{(2\mu U)^{3/2}}{3N/2} \right)^N} \frac{\left( e^{3/2} \right)^N}{\sqrt{2\pi N} \sqrt{3\pi N}} \]

\[ \approx \left[ \left( \frac{4\pi \mu U}{3h^2 N} \right)^{3/2} \frac{V}{N} \right]^N \left( e^{3/2} \right)^N \frac{1}{\sqrt{6\pi N}} \]

\[ \text{"merely large"} \]

\[ \Rightarrow S = k_B \ln \Omega_N \]

\[ = N k_B \ln \left( \frac{V}{N} \left( \frac{4\pi \mu U}{3h^2 N} \right)^{3/2} + \frac{S}{2} \right) \]