Physics 340  
Homework #7  
due Tuesday, 25 October 2011

Turn in solutions to the following problems at the beginning of class on Tuesday. As usual, if you use outside sources or computer software to help you in your solution, note them accordingly. Include any C++ or Mathematica or Maple code that you used, together with its output. Problems marked with an asterisk (*) may involve a bit more work than the others and so are worth twice as many points.

(1) We saw that one way to write the Lagrangian for a charge $q$ moving in a uniform magnetic field $\vec{B} = B\hat{z}$ is

$$\mathcal{L} = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + qBx\dot{y}.$$  

Let us imagine that the particle is confined to move in the $xy$ plane, so that we can ignore the $\dot{z}$ term above.

(a) Derive the Hamiltonian $\mathcal{H}$ and Hamilton’s equations of motion for this system.

(b) Find a solution for the equations of motion in which the particle moves in a circle of radius $a$ about the origin. (Hint: The angular velocity will be related to $m$, $q$ and $B$.)

(c) Since $\mathcal{L}$ (and thus $\mathcal{H}$) does not depend on $y$, the momentum $p_y$ is conserved. Confirm that $p_y$ is conserved for the trajectory in part (b).

(2) Do problem 13.5 in the text.

(3) A particle of mass $m$ moves along the surface of a sphere of radius $R$, with no forces acting except the forces of constraint keeping it on the sphere. Using spherical coordinates $\theta$ and $\phi$, find the Lagrangian $\mathcal{L}$, the momenta $p_\theta$ and $p_\phi$, Hamilton’s function, the Hamiltonian, and the Hamiltonian equations of motion. Show that “lines of longitude” ($\phi = \text{const}$) are possible trajectories. How about “lines of latitude” ($\theta = \text{const}$)?

(4) Do problem 13.23 in the text.

(5) The angular momentum of a particle moving in 3-D is $\vec{L} = \vec{r} \times \vec{p}$. Calculate the Poisson bracket $\{L_x, L_y\}$.  

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