Physics 340
Homework #2
due Thursday, 8 September 2011

Turn in solutions to the following problems at the beginning of class on Thursday. As usual, if you use outside sources or computer software to help you in your solution, note them accordingly. Include any C++ or Mathematica or Maple code that you used, together with its output.

(1) Do problem 3.2 in Taylor. Easy.

(2) Do problem 3.4 in Taylor. This is slightly trickier.

(3) Do problem 3.8 in Taylor. Another easy one.

(4) Do problem 3.20 in Taylor. This is indeed a “beautiful result”.

(5) Here is an old chestnut of a Physics 140 problem. A ball of mass $m$ and radius $R$ is placed inside a larger hollow sphere with the same mass $m$ and inside radius $2R$. The combination is at rest on a frictionless surface in the position shown in the diagram below. The smaller ball is released, rolls around inside the hollow sphere, and finally comes to rest at the bottom. How far will the larger sphere have moved during this process?

(6) Start with a uniform disk of material of radius $a$ and total mass $M$. Cut out a “pie slice”, a wedge-shaped piece having a vertex angle $\alpha$. For convenience, you may suppose one edge of this slice lies along the $x$-axis. (a) Find the center of mass of the pie slice. (b) Find the center of mass of the rest of the pie.
(7) Do problem 3.34 in Taylor. (Be sure to remember this calculation next time you juggle a burning torch.)

(8) You have heard about the idea of an “elastic” collision. This problem develops an alternate approach that does not depend on the idea of kinetic energy.

Consider two masses. We can write the velocity of each one as \( \vec{v}_\alpha = \vec{V} + \vec{v}_\alpha \), where \( \vec{v}_\alpha \) is the velocity of particle \( \alpha \) relative to the center of mass. (Note: \( \alpha = 1, 2 \).)

(a) Show that \( m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0 \).

(b) Suppose the two masses collide. Let “i” and “f” indicate initial and final conditions—that is, conditions before and after the collision. Show that \( v_{\alpha f} = \kappa v_{\alpha i} \), with the same constant \( \kappa \) for both particles. (The value of \( \kappa \) is a property of the collision process.)

(c) Collisions in which \( \kappa = 1 \) are called elastic collisions. The non-elastic ones can be divided into two kinds: subelastic (\( \kappa < 1 \)) and superelastic (\( \kappa > 1 \)). Describe what happens in a completely subelastic collision, one in which \( \kappa = 0 \) (the smallest possible value).

(d) Now let’s make the familiar connection to kinetic energy. The kinetic energy of particle \( \alpha \) is defined to be

\[ K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2. \]

(Note that this \( v_\alpha \) is the total speed, not the speed relative to the CM.) Show that the total kinetic energy \( K_1 + K_2 \) is unchanged in an elastic collision, decreased in a subelastic collision, and increased in a superelastic collision.