7.41 \ A \ \text{is spontaneous decay probability per unit time.} \\
B u(f) \ \text{is probability of absorption per unit time.} \\
B' u(f) \ \text{is probability of stimulated transition per unit time.} \\
a) \quad \frac{dN_1}{dt} = AN_2 + B' u(f) N_2 - Bu(f) N_1 \\
b) \quad \frac{dN_1}{dt} = 0 \\
\Rightarrow \quad [A + B' u(f)] N_2 = Bu(f) N_1 \Rightarrow \quad \left[ A + B' u(f) \right] N_2 = B u(f) N_1 \\
H, \ N_2 \ \text{and} \ N_1 \ \text{are related by Boltzmann factor, then} \\
\frac{N_2}{N_1} = e^{-\Delta E/kT} = e^{\epsilon/kT} \\
\Rightarrow \quad N_2 = e^{-\epsilon/kT} N_1 \\
\Rightarrow \quad \left[ A + B' u(f) \right] e^{-\epsilon/kT} N_1 = Bu(f) N_1 \\
\left[ A + B' u(f) \right] = B e^{\epsilon/kT} u(f) \\
A = \left[ B e^{\epsilon/kT} - B' \right] u(f)
What is $u(f)$?

\[ \int_{f_1}^{f_2} u(f) \, df \] is energy between two frequencies.

We have a relation for $u(\varepsilon)$ defined by

\[ \int_{\varepsilon_1}^{\varepsilon_2} u(\varepsilon) \, d\varepsilon \]

They are related by change of variable, $\varepsilon = hf$

\[ u(f) = h \cdot u(\varepsilon) \quad \text{since} \quad d\varepsilon = h \, df \]

\[ u(\varepsilon) = \frac{8\pi h}{(hc)^3} \frac{(hf)^3}{e^{hf/kT} - 1} = \frac{8\pi h}{e^{hf/kT} - 1} \]

\[ A = \left[ B \, e^{-\frac{hf}{kT}} \right] \frac{8\pi h (f/c)^3}{e^{hf/kT} - 1} \]

Strock! But $A$ can't be temperature dependent since it is an intrinsic property of atom

\[ B' = B \quad \text{(existence of stimulated emission!)} \]

and $A = B \times 8\pi h (f/c)^3$

\[ \Rightarrow \frac{A}{B} = 8\pi h (f/c)^3 \]