7.22 a) Equation 7.34 and 7.35 remain intact. But we must replace 7.36 with

$$E = pc = \frac{hc}{2L} \sqrt{\frac{1}{\hbar^2 + n_1^2 + n_2^2}} = \frac{hc}{2L} n$$

where \( n \) is the length of the \( \vec{n} \) in \( n \)-space.

Eqn 7.37 now must be revised

$$E_F = \frac{hc}{2L} n_{\text{max}}.$$

But Eqn 7.38 hasn't changed. So

$$N = \frac{\pi n_{\text{max}}^3}{3} \Rightarrow n_{\text{max}} = \left( \frac{3N}{\pi} \right)^{\frac{1}{3}}$$

$$\Rightarrow E_F = \frac{hc}{2L} \left( \frac{3N}{\pi} \right)^{\frac{1}{2}} = hc \left( \frac{3 N}{8 \pi L^3} \right)^{\frac{1}{2}}$$

$$E_F = hc \left( \frac{3}{3 \pi \sqrt{V}} \right)^{\frac{1}{2}}$$

as expected.

b) To calculate the average energy, we can start with equation 7.42, but use our new relationship for \( E(n) \):
\[ U = \pi \int_0^{n_{\text{max}}} \varepsilon(n) n^2 \, dn \]

\[ = \frac{\pi \hbar c}{2L} \int_0^{n_{\text{max}}} n^3 \, dn \]

\[ = \frac{\pi \hbar c}{8L} n_{\text{max}}^4 \]

\[ U = \frac{\pi \hbar c}{8L} \left( \frac{3N}{\pi} \right)^{4/3} \]

\[ \Rightarrow U = \frac{3N \hbar c}{4} \left( \frac{3N}{8\pi} \right)^{1/3} = \frac{3}{4} N \varepsilon_F \]