6.39 a) From equation 6.55 we have
\[ P(N > N_0) = \frac{4}{\pi N^2} \int_{N_0}^{\infty} x^2 e^{-x^2} \, dx \]
where \( x = \frac{N_0}{N_{\text{max}}} \)

when \( N_{\text{max}} = \sqrt{\frac{2kT}{m}} \) is the most probable velocity.

For \( N_2 \), 
\[ m = \frac{28 \times 10^{-3} \text{kg}}{6.02 \times 10^{23}} = 4.65 \times 10^{-26} \text{ kg} \]

\[ N_{\text{max}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \text{ J/}K \times 1000 \text{ K}}{4.65 \times 10^{-26} \text{ kg}}} \]

\[ = 770 \text{ m/s} \]

\[ x = \frac{11 \times 10^3 \text{ m/s}}{770 \text{ m/s}} = 14.28 \]

So now need \( \int_{14.28}^{\infty} x^2 e^{-x^2} \, dx \).

I used Maple to evaluate this integral (see attached sheet). The result for the probability of a \( N_2 \) molecule having a speed larger than the escape velocity is \( 4 \times 10^{-87} \), so fantastically small.
61. For hydrogen $\frac{m_{H_2}}{m_{N_2}} = \frac{2}{28} \Rightarrow v_{max, H_2} = \sqrt{\frac{28}{2}} = 3.74$

$\Rightarrow v_{max, H_2} = 3.74 \cdot 770 \text{ m/s} = 2880 \text{ m/s}$

$\Rightarrow x_{H_2} = \frac{11 \times 10^3}{2880} = 3.82$

$\Rightarrow \frac{4}{\sqrt{\pi}} \int_{-3.82}^{0} x^2 e^{-x^2} \, dx = 0.2 \times 10^{-5}$ (See Maple)

For helium: $\frac{m_{He}}{m_{N_2}} = \frac{4}{28} \Rightarrow v_{max, He} = 770 \cdot \sqrt{\frac{28}{4}}$

$= 2040 \text{ m/s}$

$\Rightarrow x_{He} = \frac{11 \times 10^3}{2040} = 5.40$

$\Rightarrow \frac{4}{\sqrt{\pi}} \int_{-5.40}^{0} x^2 e^{-x^2} \, dx = 0.13 \times 10^{-11}$

So probability for escape is much higher for H$_2$ and He than for N$_2$, and it is a strong function of mass. Explain why there is no H$_2$ or He in Earth's atmosphere. If there had been some in the past, it would have escaped by now.
c) Let's repeat the calculation for $N_2$, but now with respect to the Moon's escape speed of 2.4 km/s.

$$\frac{2.4 \times 10^3}{1770 \text{ m/s}} = 3.12$$

$$\Rightarrow \text{escape } = \frac{4}{\sqrt{\pi}} \int_{3.12}^{\infty} x^2 e^{-x^2} dx = 2 \times 10^{-4}$$

So even $N_2$ would have long ago escaped from the Moon's gravitational pull.
Schroeder, problem 6.39

> restart;

First calculate probability that velocity is larger than the escape velocity for N_2

> f(x) := (x^2) * (exp(-x^2));

\[
f(x) = x^2 e^{-x^2}
\]

> int(f(x), x=14.28..infinity);

\[
\int f(x) \, dx = 0.196845148 \times 10^{-47}
\]

> evalf(4*%/sqrt(Pi));

\[
0.4442304993 \times 10^{-47}
\]

repeat for H_2

> int(f(x), x=3.82..infinity);

\[
\int f(x) \, dx = 0.9074370995 \times 10^{-6}
\]

> evalf(4*%/sqrt(Pi));

\[
0.2047866237 \times 10^{-5}
\]

repeat for He

> int(f(x), x=5.40..infinity);

\[
\int f(x) \, dx = 0.5951147425 \times 10^{-12}
\]

> evalf(4*%/sqrt(Pi));

\[
0.1343030155 \times 10^{-11}
\]

repeat for N_2 on Moon

> int(f(x), x=3.12..infinity);

\[
\int f(x) \, dx = 0.00009690057540
\]

> evalf(4*%/sqrt(Pi));

\[
0.0002186811811
\]