5.55 a) If we are looking to expand \( p(V) \), then start with expression in form (in reduced variables, see problem 5.51)

\[
p = \frac{8t}{3v} - \frac{3}{v^2} - 8t (3v-1)^{-1} - 3v^{-2}
\]

We are expanding \( p \) in Taylor series around \((P_c, V_c)\)

\[
p = p(V_c) + \frac{dp}{dv}(V-V_c) + \frac{1}{2!} \left( \frac{d^2p}{dv^2} \right)_{V=V_c} (V-V_c)^2 + \ldots
\]

\[
= \frac{1}{3!} \left( \frac{d^3p}{dv^3} \right)_{V=V_c} (V-V_c)^3 + \ldots
\]

\[
\frac{dp}{dv} = -8t (3v-1)^{-2} \cdot 3 + 6v^{-3} = -24t (3v-1)^{-2} + 6v^{-3}
\]

\[
\frac{d^2p}{dv^2} = 48t (3v-1)^{-3} (3) - 18v^{-4} = 144t (3v-1)^{-3} - 18v^{-4}
\]

\[
\frac{d^3p}{dv^3} = -432t (3v-1)^{-4} \cdot 3 + 72v^{-5} = -1296t (3v-1)^{-4} + 72v^{-5}
\]

Now evaluate each of these derivatives at \( V = V_c = 1 \).
\[
\left( \frac{dp}{dV} \right)_{v=1} = -24t + 6 = -6t + 6 = -6(t-1)
\]

\[
\left( \frac{d^2p}{dv^2} \right)_{v=1} = \frac{144t - 18}{2^3} = 18(t-1)
\]

\[
\left( \frac{d^3p}{dv^3} \right)_{v=1} = \frac{-1296t + 72}{2^4} = -81t + 72 = 9(9(t-8))
\]

\[p(v) = p(1) = \frac{8t - 3}{2} = (4t - 3)
\]

\[\Rightarrow p = (4t - 3) - 6(t-1)(n-1) + \frac{1}{2} 18(t-1)(n-1)^2
\]

\[-\frac{9}{6} (9(t-8)(n-1)^3 + \ldots
\]

\[p = (4t - 3) - 6(t-1)(n-1) + 9(t-1)(n-1)^2 - \frac{3}{2} (9(t-8)(n-1)^3
\]

If \((t-1)\) is small then can replace \(t=1\) in expression other than \((t-1)\)

\[(p-1) = -6(t-1)(n-1) + 9(t-1)(n-1)^2 - \frac{3}{2} (n-1)^3
\]

So \((p-1)\) in cubic in \((n-1)\) for fixed \((t-1)\).
I see no reason to neglect quadratic term, but it is symmetric about \( N=1 \), so doesn't affect structure of isotherm. So will neglect it based on Schroedlin's advice.

\[
(p-1) = -6 (t-1) (N-1) - \frac{3}{2} (N-1)^2
\]

b) Consider the situation above the critical temperature \( (t-1) > 0 \), then as \( N \to 1 \)
\[p \to 1.\] Below critical temperature the graph looks like

\[
\text{But note that Maxwell's constraint gives } p=1 \text{ here as well. (Could put back } p=4t+3 \text{ to restore } t \text{ dependence). } \Rightarrow \frac{dp}{dt} = 4
\]

c) \( p-1 = 0 \) \[\Rightarrow -6 (t-1) (N-1) = \frac{3}{2} (N-1)^2 \]

\[-4 (t-1) = (N-1)^2\]

\[\Rightarrow (N-1) = \sqrt{4(1-t)} = 2(1-t)^{1/2} \Rightarrow \beta = \frac{1}{2}\]

for van der Waals model.
d) From eqn 5.47 \[ L = T \Delta V \left( \frac{dP}{dT} \right) \]

\[ T = T_c t, \quad \Delta V = 2V_c (v-1) \]
\[ dT = T_c dt \quad P = p \tilde{P}_c \Rightarrow dP = \tilde{P}_c dp \]

\[ \Rightarrow \frac{dP}{dT} = \frac{\tilde{P}_c}{T_c} \left( \frac{dp}{dt} \right) \]

\[ L = 2 \frac{T_c \tilde{V}_c \tilde{P}_c}{T_c} (v-1) \]

\[ L = 8 \tilde{V}_c \tilde{P}_c (v-1) = (8) \left( 3N_b \right) \left( \frac{1}{27} \frac{\alpha}{b^2} \right)^L = \frac{24 \alpha}{27} \frac{N_a}{b} (v-1) \]

\[ L = \frac{24 \alpha}{27} \frac{N_a}{b} \left( 2 (1-t)^\frac{1}{2} \right) \]
\[ L = \frac{48 \alpha}{27} \frac{N_a}{b} (1-t)^{\frac{1}{2}} \Rightarrow \]

\[ \text{square root dependence} \]

\[ \text{Lagrange above } T_c. \]

\[ (1-t) \]

e) From above \( (p-1) = -b (t-1) (v-1) - \frac{3}{2} (v-1)^2 \), but at \( T=T_c, \quad t-1 = 0 \)

\[ \Rightarrow p-1 = -\frac{3}{2} (v-1)^3 \Rightarrow J = 3 \text{ for v.d.W.} \]