4.14 a) An efficiency, or COP, is defined by COP = benefit/cost. Here the benefit is the heating of the hot reservoir and the cost is the amount of work it takes. So COP = \( \frac{Q_{\text{in}}}{W} \).

b) \( Q_{\text{in}} = Q_{\text{c}} + W \).

From the first law we can write \( W = Q_{\text{in}} - Q_{\text{c}} \).

\[ \Rightarrow \text{COP} = \frac{Q_{\text{in}}}{Q_{\text{c}}} = \frac{1}{1 - \frac{Q_{\text{c}}}{Q_{\text{in}}}} = \frac{1}{1 - \frac{Q_{\text{c}}}{Q_{\text{in}}}} \]

\( Q_{\text{c}} \) and \( Q_{\text{in}} > 0 \), so, yes, in general COP > 1. (always).

c) In the ideal limit there is no change of entropy so \( \frac{Q_{\text{c}}}{T_{\text{c}}} = \frac{Q_{\text{in}}}{T_{\text{in}}} \Rightarrow \frac{Q_{\text{c}}}{Q_{\text{in}}} = \frac{T_{\text{c}}}{T_{\text{in}}} \)

\[ \Rightarrow \text{COP}_{\text{ideal}} = \frac{1}{1 - \frac{T_{\text{c}}}{T_{\text{in}}}} \]

In the non-ideal case \( Q_{\text{c}}/T_{\text{c}} < Q_{\text{in}}/T_{\text{in}} \)

\[ \Rightarrow \frac{Q_{\text{c}}}{Q_{\text{in}}} < \frac{T_{\text{c}}}{T_{\text{in}}} \Rightarrow 1 - \frac{Q_{\text{c}}}{Q_{\text{in}}} > 1 - \frac{T_{\text{c}}}{T_{\text{in}}} \]

\[ \Rightarrow \text{COP} < \frac{1}{1 - \frac{T_{\text{c}}}{T_{\text{in}}}} \]

d) Well the COP for a furnace is benefit/cost = 1. For heat pump COP > 1.
To put some numbers to this, let's estimate $T_c$ and $T_H$. Say $T_c = 20^\circ F = -6.67^\circ C = 266.6\ K$. Now let $T_H = 75^\circ F = 297\ K$. So

$$\text{COP}_{\text{max}} = \frac{1}{1 - 266/297} = 9.6$$

So it looks like a great advantage.