3.37 a) If the gas atoms have a potential energy as well as kinetic, then
\[ U = U_{\text{kin}} + U_{\text{pot}} \]
If each molecule has potential energy \( mgz \), then
\[ U_{\text{pot}} = Nmgz \]
So, by equation 3.60
\[ \mu = \left( \frac{\partial U}{\partial N} \right)_{S,V} = \left( \frac{\partial U_{\text{kin}}}{\partial N} \right)_{S,V} + \left( \frac{\partial U_{\text{pot}}}{\partial N} \right)_{S,V} \]
\[ = \mu(z=0) + mgz \quad \text{since } \left( \frac{\partial U_{\text{kin}}}{\partial N} \right)_{S,V} \]
\[ \text{is just what we deal with } z=0, \text{ i.e. all kinetic energy.} \]
But \( \mu(z=0) \) is just given by eqs 3.43,
\[ \mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2\pi m kT}{\hbar^2} \right)^{3/2} \right] + mgz \]
as was to be shown.

Alternatively we can re-examine the derivation of the Sackur-Tetrode equation. The \( U \) in the ST equation comes only from the kinetic energy (since it determines the volume in momentum space). Thus, if \( U = U_{\text{kin}} + U_{\text{pot}} \) we should substitute \( U_{\text{pot}} \) for \( U \) in the ST equation.
\[ S = Nk \left[ \ln \left( \frac{V}{(2\pi m kT)^{3/2}} \right) - \ln N \frac{3z^2}{2} + \frac{5z}{2} \right] \]
(from eq. 3.62)
\[ S = Nk \left[ \ln \left( \frac{4\pi m}{3h^2} \right)^{3/2} + \frac{1}{2} \ln (U - U_{pot})^{5/4} - \ln N^{5/4} + \frac{5}{2} \right] \]

only new term that depends on \( N \)

\[ \left( \frac{2S}{\partial N} \right)_{u,v} = \mu(z=0) - NkT \frac{2}{\partial N} \left( \ln (U - U_{pot})^{5/4} \right) \]

\[ = \mu(z=0) - NkT \frac{1}{(U - U_{pot})^{3/4}} \left( \frac{3}{2} (U - U_{pot})^{1/4} \right) \frac{2U_{pot}}{\partial N} \]

\[ \mu(z) = \mu(z=0) - \frac{3}{2} NkT \frac{1}{(U - U_{pot})} \left( -mgz \right) \]

But \( U - U_{pot} = U \text{kin} = \frac{3}{2} NkT \)

\[ \Rightarrow \mu(z) = \mu(z=0) + mgz. \]

b) Assuming the two "chunks" are in diffusion equilibrium implies their chemical potentials are equal.

\[ \Rightarrow \mu(z) = \mu(0) \]

\[ -kT \ln \left[ \frac{V}{N(z)} \left( \frac{2\pi m kT}{h^2} \right)^{3/2} \right] + mgz = \]

\[ -kT \ln \left[ \frac{V}{N(0)} \left( \frac{2\pi m kT}{h^2} \right)^{3/2} \right] \]
\[ + kT \ln N(\varepsilon) + mg \varepsilon = kT \ln N(0) \]

\[ kT \ln \left( \frac{N(\varepsilon)}{N(0)} \right) = -mg \varepsilon \]

\[ \ln \left( \frac{N(\varepsilon)}{N(0)} \right) = - \frac{mg \varepsilon}{kT} \]

\[ \Rightarrow N(\varepsilon) = N(0) e^{-mg \varepsilon / kT} \]