3.34  a) This system is analogous to the two state paramagnet. \( S = k \ln \left( \frac{N}{N_R} \right) \).

If we assume \( N, N_R, N_L \) are all separately large, then we can use Stirling's approximation to simplify this result:

\[
\frac{S}{k} = \ln N! - \ln N_R! - \ln N_L!
= N \ln N - N - N_R \ln N_R + N_R - N_L \ln N_L + N_L
\]

\[
\frac{S}{k} = N \ln N - N_R \ln N_R - (N-N_R) \ln (N-N_R)
\]

b) \( L = l (N_R - N_L) = l \left( 2N_R - N \right) \)

\[
\Rightarrow \frac{L}{l} = 2N_R - N \Rightarrow 2N_R = N + \frac{l}{l}
\]

\[
N_R = \frac{N}{2} + \frac{L}{2L} = \frac{N}{2} \left( 1 + \frac{L}{N_L} \right)
\]

c) By the first law of thermodynamics
\( dU = Q + W \). Here \( W = FdL \) and in usual \( Q = TdS \). So, \( dU = TdS + FdL \)

d) If we consider a process at constant energy, we can derive from part c) that
\( dU = 0 = TdS + FdL \)

\[
\Rightarrow F = -T \left( \frac{\partial S}{\partial L} \right)_U
\]

We have an expression for the entropy in terms of \( N_R \) and we have a relationship between \( L \) and \( N_R \). Thus we can use the chain rule.
\[ \frac{\partial S}{\partial L} = \frac{\partial S}{\partial N_R} \cdot \frac{\partial N_R}{\partial L} \quad \frac{\partial N_R}{\partial L} = 2L \text{ from b)} \]

\[ \frac{\partial S}{\partial L} = \frac{k}{2L} \left( \frac{-N_R - \ln N_R + (N-N_R)}{N_R} \cdot \ln (N-N_R) \right) \]

\[ = \frac{k}{2L} \ln \left( \frac{N-N_R}{N_R} \right) \]

\[ \frac{\partial S}{\partial L} = \frac{k}{2L} \ln \left( \frac{1-L/N_L}{1+L/N_L} \right) \]

\[ F = -T \left( \frac{\partial S}{\partial L} \right)_{N} = -\frac{kT}{2L} \ln \left( \frac{1-L/N_L}{1+L/N_L} \right) \]

\[ F = \frac{kT}{2L} \ln \left( \frac{1+L/N_L}{1-L/N_L} \right) \]

(But how can we tell that \( u \) is constant in how we took derivative?)

e) If \( L \ll N_L \) (high temperature limit)

\[ F = \frac{kT}{2L} \left[ \ln \left( 1+\frac{L}{N_L} \right) - \ln \left( 1-\frac{L}{N_L} \right) \right] \approx \frac{kT}{2L} \left[ \frac{L}{N_L} + \frac{L}{N_L} \right] = \frac{kT}{N_L^2} L \]
f) From d) and e) we can see that if we hold length fixed and increase temperature, then the tension is going to increase. So adding heat tends to decrease length. This makes sense, since adding heat should increase entropy which tends to mean more equal numbers of $N_s$ and $N_c$.

g) If you stretch the rubber band, you obviously increase its length. From part a) and b) we can see that this decreases entropy of the chain of links. But if you do this quickly then you might imagine that this in an adiabatic process $\Rightarrow Q=0. \Rightarrow TdS=0$. This would be a problem except for the hint that there is a vibrational component to the total entropy of the rubber band. So if $dS=0$ then the loss of entropy in the links must be made up in the vibrational entropy. From our experience with vibrational systems, an increase in vibrational entropy is associated with an increase in temperature $\Rightarrow$ stretching rubber band causes it to warm up.