3.33 Imagine a process by which we add a small amount of heat $Q$, changing the temperature by an amount $\Delta T$, while keeping the volume fixed. The thermodynamic identity says

$$dU = T\,dS - P\,dV$$

Since the volume is held fixed, $dV = 0$, so

$$dU = T\,dS$$

By definition, $dU = (\frac{\partial U}{\partial T})_V \Delta T$, but by eqn 1.44 $C_v = (\frac{\partial U}{\partial T})_V$. So $dU = C_v \Delta T$

By definition as well, $dS = (\frac{\partial S}{\partial T})_V \Delta T$. So, substituting these into the thermodynamic identity, we see

$$C_v \Delta T = T \left( \frac{\partial S}{\partial T} \right)_V \Delta T$$

Thus

$$\Rightarrow C_v = T \left( \frac{\partial S}{\partial T} \right)_V$$

From eqns 1.53, we have

$$dH = dU + P\,dV$$

But from the thermodynamic identity (3.46)

$$dU = T\,dS - P\,dV$$

So

$$dH = T\,dS - P\,dV + P\,dV = T\,dS$$
If we add heat to change the temperature by \( \Delta T \) while keeping the pressure constant,

\[
\Delta H = \left( \frac{\partial H}{\partial T} \right)_P \Delta T = C_p \Delta T
\]

\[
= T \left( \frac{\partial S}{\partial T} \right)_P \Delta T
\]

\[
\Rightarrow C_p = T \left( \frac{\partial S}{\partial T} \right)_P
\]