

1.8 a) For steel, we are given that $\alpha = 1.1 \times 10^{-5} \text{ K}^{-1}$. Using the definition $\alpha = \frac{(\Delta L / L)}{\Delta T}$,

we can see that $\Delta L = \alpha L \Delta T$. If we estimate the temperature difference between a hot summer day and a cold winter night as about 50°C , then

$\Delta L = (1.1 \times 10^{-5} \text{ K}^{-1}) \cdot (50 \text{ K}) \cdot (10^3 \text{ m}) = 0.55 \text{ m}$. That is about 20" and makes you wonder how to design a bridge that can deal with that much variation in length.

b) If one piece of metal grows longer than the other then the strip will tend to bend or curl in the direction of the metal with the smaller thermal expansion coefficient. If one attaches a pointer to the coil, you can use it to indicate temperature.

c) Let the original dimensions of an object be x, y, z . Imagine they expand by an amount $\Delta x, \Delta y, \Delta z$. The change in volume is then

$$\begin{aligned}\Delta V &= (x + \Delta x)(y + \Delta y)(z + \Delta z) - xyz, \\ \Delta V &= xyz + xy\Delta z + xz\Delta y + yz\Delta x + O(\Delta^2) - xyz, \\ \Delta V &= xy\Delta z + xz\Delta y + yz\Delta x + O(\Delta^2),\end{aligned}$$

where $O(\Delta^2)$ represents terms that involve products of at least two small quantities, which we now neglect. Thus,

$$\begin{aligned}\Delta V / V &= (xy\Delta z + xz\Delta y + yz\Delta x) / xyz, \\ \Delta V / V &= \Delta z / z + \Delta y / y + \Delta x / x.\end{aligned}$$

Finally, dividing through by ΔT , we get

$$(1/V)(\Delta V / \Delta T) = (1/z)(\Delta z / \Delta T) + (1/y)(\Delta y / \Delta T) + (1/x)(\Delta x / \Delta T).$$

Or, using the definitions of volume and linear expansion coefficients, this equation becomes $\mathbf{b} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, which was to be shown.