1.34 An ideal diatomic gas in the approximation that rotational degrees of freedom are active and vibrational degrees of freedom are frozen out has $f = 5$ degrees of freedom. This will allow us to use $U_{thermal} = N \cdot f \cdot (1/2kT) = f/2PV$ to calculate the internal energy of the gas. (Note the odd numbering of the steps!)

a) Step A: Since $W = -P\Delta V$ and the change in volume is zero, there is no work done on the gas in step A. So, $W = 0$. Using the hint, we now calculate the change in internal energy, before calculating Q. The internal energy of the gas at the start of step A is $U_{thermal} = 5/2P_1V_1$ and at the end $U_{thermal} = 5/2P_2V_1$, so $\Delta U = 5/2 \cdot (P_2 - P_1) \cdot V_1$. Now we can use the first law to calculate the heat energy that was absorbed by the gas. Since $\Delta U = Q + W$, we have $Q = \Delta U - W$. So, $Q = \Delta U - 0 = 5/2 \cdot (P_2 - P_1) \cdot V_1$.

Step B: $W = -P\Delta V = -P_2 \cdot (V_2 - V_1)$. The change in internal energy is $\Delta U = 5/2 \cdot P_2 \cdot (V_2 - V_1)$. Using the first law, we get $Q = \Delta U - W = 5/2 \cdot P_2 \cdot (V_2 - V_1) + P_2 \cdot (V_2 - V_1) = 7/2 \cdot P_2 \cdot (V_2 - V_1)$.

Step D: $W = 0$. $\Delta U = 5/2 \cdot (P_1 - P_2) \cdot V_2$. Then, $Q = \Delta U - 0 = 5/2 \cdot (P_1 - P_2) \cdot V_2$.

Step C: $W = -P\Delta V = -P_1 \cdot (V_1 - V_2)$. $\Delta U = 5/2 \cdot P_1 \cdot (V_1 - V_2)$.

$Q = \Delta U - W = 5/2 \cdot P_1 \cdot (V_1 - V_2) + P_1 \cdot (V_1 - V_2) = 7/2 \cdot P_1 \cdot (V_1 - V_2)$

b) Step A: The volume is held fixed, so no work is being done, but the pressure and hence internal energy is going up. So energy is the form of heat must be being added to the gas during the process. This is confirmed by the sign of Q in the calculation above.

Step B: The volume of the container is getting larger, so the gas is doing work. If this were being done with no heat being added, we would expect the pressure to fall, but it is not. So, the gas must be absorbing heat energy in the process. This is confirmed in the calculation above.

Step D: The pressure is falling even though the container is staying at fixed volume. So heat energy must be being taken out of the gas. Since $(P_1 - P_2) < 0$, we see that this agrees with the calculation above.

Step C: Work is being done on the gas, so $W > 0$ as seen in the calculation (since $(P_1 - P_2) < 0$). But the pressure is staying the same, so heat must also be being removed from the gas. This agrees with the calculation above, since $(V_1 - V_2) < 0$.

c) Work: $\sum W = W_A + W_B + W_D + W_C = 0 - P_2 \cdot (V_2 - V_1) + 0 - P_1 \cdot (V_1 - V_2)$

$\sum W = -(P_2 - P_1) \cdot (V_2 - V_1)$, or, just the negative of the area of the PV diagram enclosed by the lines describing the process.

Internal energy: This had better be zero, but let's check.

$$\sum \Delta U = \Delta U_A + \Delta U_B + \Delta U_D + \Delta U_C$$

$$\sum \Delta U = 5/2 \cdot (P_2 - P_1) \cdot V_1 + 5/2 \cdot P_2 \cdot (V_2 - V_1) + 5/2 \cdot (P_1 - P_2) \cdot V_2 + 5/2 \cdot P_1 \cdot (V_1 - V_2)$$

$\sum \Delta U = 5/2 \cdot (P_2 - P_1) \cdot (V_1 - V_2) + 5/2 \cdot (P_1 - P_2) \cdot (V_1 - V_2) = 0$. As expected.

<u>Heat</u>: $\sum Q = Q_A + Q_B + Q_D + Q_C$

$$\sum Q = 5/2 \cdot (P_2 - P_1) \cdot V_1 + 7/2 \cdot P_2 \cdot (V_2 - V_1) + 5/2 \cdot (P_1 - P_2) \cdot V_2 + 7/2 \cdot P_1 \cdot (V_1 - V_2)$$

$$\sum Q = -5/2 \cdot (P_2 - P_1) \cdot (V_2 - V_1) + 7/2 \cdot (P_2 - P_1) \cdot (V_2 - V_1)$$

$$\sum Q = (P_2 - P_1) \cdot (V_2 - V_1)\,.$$

This is just the negative of $\sum W$ as we could have calculated immediately from the first law (but it is always nice to check these things!) So this is as expected as well.