1.16 a) Let the density of air be $r(z)$. Then, for a slab of air of area A and thickness dz to be in mechanical equilibrium, the forces must add vectorially to zero. The force acting in the upward directions is $P(z)A$. The forces acting in the downward direction are $P(z+dz)A$ and the weight $mg = rVg = rAdzg$. Thus, for equilibrium:

$$P(z)A - P(z+dz)A - rAdzg = 0,$$

$$(P(z+dz) - P(z)) = -r\,dzg$$

$$\frac{dP}{dz} = -rg\,.$$

b) We can use the ideal gas law to determine an expression for r.

$$PV = NkT$$

$$\frac{N}{V} = \frac{P}{kT}$$

$$r = m\frac{N}{V} = m\frac{P}{kT},$$

where m is the mass of a molecule of air. Inserting this equation into the differential equation in part a), we see

$$\frac{dP}{dz} = -rg = -\frac{mg}{kT}P\,.$$

As we were supposed to find.

c) Solve this equation by separation of variables,

$$\frac{dP}{dz} = -\frac{mg}{kT}P$$

$$\frac{dP}{P} = -\frac{mg}{kT}dz$$

$$\int_{P_0}^{P}\frac{dP}{P} = -\frac{mg}{kT}\int_0^z dz$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{mg}{kT}z$$

$$P = P_0\exp\left(-\frac{mg}{kT}z\right).$$

Part b) can be used to relate the density and the pressure. We can multiply the last equation on both side by m/kT, to get

$$\frac{mP}{kT} = \frac{mP_0}{kT}\exp\left(-\frac{mg}{kT}z\right)$$

Comparing this to the expression for the density in part b) , see that

$$r = r_0\exp\left(-\frac{mg}{kT}z\right).$$

d) It is useful to start by calculating the quantity $\frac{mg}{kT}$, since it will be used in all of the calculations. If we use 29 as the average molecular weight of the atmosphere, then the average molecule weighs $29\times10^{-3}\ \text{kg}/6.02\times10^{23} = 4.82\times10^{-26}\ \text{kg}$. Now $kT = (1.38\times10^{-23}\ \text{J/K})\cdot(300\ \text{K}) = 4.14\times10^{-21}\ \text{J}$. Thus $\frac{mg}{kT} = 1.14\times10^{-4}\ \text{m}^{-1}$.

So now we can plug in altitudes to find:

Ogden, UT: z = 1430 m, $P = (1\ \text{atm})\exp(-(1.14\times10^{-4}\ \text{m}^{-1})\cdot(1430\ \text{m})) = 0.85\ \text{atm}$.
Leadville, CO: z = 3090 m, P = 0.70 atm.
Mt. Whitney, CA: z = 8840 m, P = 0.60 atm.
Mt. Everest, Nepal: z = 8840 m, P = 0.37 atm.