

Data Fitting and Modeling Radioactive Decay

This introductory exercise has a couple of purposes:

- Refresh (or introduce) the concepts of data fitting and extracting information from the mathematical fits of the data.
- Refresh (or introduce) the use of Microsoft Excel
- Present the concept of exponential decay

To do this, you will need a data set. Rolling 20-sided dice under various rules and counting “survivors” will create this data set.

NOTE: 20-sided dice bounce around a lot. This has both pros and cons:

- **Pro: They “randomize” very readily, so they don’t need much of a roll to produce the desired random results.**
- **Con: They easily bounce off tables and onto the floor.**
- **Additionally: The little buggers are expensive. SO...please roll them gently and keep track of them ALL. (Thank you.)**

Linear Model:

In the first exercise, you will generate a linear data set.

1. Open Excel. Put your starting number of dice (75) in the first row of your Excel spreadsheet.
2. Roll the dice (**gently**) and remove all the 1s. Count the survivors and record it in row 2 of your spreadsheet. Put the ones you removed back in with the rest of the dice.
3. Roll again and remove the 1s and the 2s. Again, record survivors and put the ones taken out back in.
4. Repeat this process for
 - a. 1s, 2s, and 3s
 - b. 1s, 2s, 3s, and 4s
 - c. 1s, 2s, 3s, 4s, and 5s
 - d. 1s, 2s, 3s, 4s, 5s, and 7s. (6s are too hard to differentiate from 9s, so just skip that one.)
 - e. 1s, 2s, 3s, 4s, 5s, 7s, and 8s
5. Graph (scatter-plot) the data in Excel. Add a trend line. Be sure to “Display Equation” on chart.

Now consider what you would expect. You started with 75 dice. On the first throw, you removed the 1s. Since there are 20 sides to the dice, the 1s constitute 5% ($1/20^{\text{th}}$) of the possible results. Hence, on average, after the first roll, you would expect the number of survivors to decrease by 5%.

$$N(1) = N(0) - 5\% * N(0) = 75 - 5\% * 75 = 75 - 3.75 = 71.25 \quad \text{Equation 1}$$

Here, $N(1)$ is the average expected number after 1 roll and $N(0)$ is the starting number of dice (the number after 0 rolls). Note that this is an average. Clearly no roll will actually end up with a fractional number of survivors!

On the second roll, you removed two dice, each of which makes up 5% of the total. Hence

$$N(2) = N(0) - 2 * 5\% * N(0) = 75 - 7.5 = 67.5 \quad \text{Equation 2}$$

...and so on. Thus, if “ x ” is the number of throws, the model for the number of survivors after x rolls is

$$N(x) = N(0) - x * 5\% * N(0) = 75 - 3.75 * x \quad \text{Equation 3}$$

This is a linear equation in x , an equation of the form

$$y(x) = a + b * x \quad \text{Equation 4}$$

where “ a ” is the intercept and “ b ” is the slope. Compare the values you found for “ a ” and “ b ” in your trend line with their expected values as listed in Equation 3. (Note the minus sign.)

The above exercise is trivial in that you already knew the number of sides of each die (20), and hence, the probability (5%) of throwing a 1. However, you can see that the process of taking data, fitting with a model-dictated equation, and analyzing the fitting coefficients (here “ a ” and “ b ”) enabled you to determine the probability of throwing a 1 without prior knowledge of the system. This is the power of data fitting.

Exponential model

In this exercise, you will model the function that describes many nuclear processes.

1. Open Excel. Put your starting number of dice (75) in the first row.
2. Roll the dice (**gently**) and remove all the 1s, 2s, 3s, and 4s. Count the “survivors” and record it in row 2. Do **not** replace the ones you removed.
3. Roll again and remove the 1s, 2s, 3s, and 4s. Record survivors and do **not** replace the ones taken out.
4. Continue this process for 15 rolls. If your dice are all gone before then, stop.
5. Graph (scatter-plot) the data in Excel. Add a linear trend line. Note the ugly fit. Note also the fact that, after a number of throws, your linear model predicts a negative number of dice remaining.
6. Make a second chart of the same data. Add an **exponential** trend line. Be sure to “Display Equation” on chart.

What would you expect? Well, it is clear that the average number of dice removed depends upon how many you have. (You expect more 1s to show in a throw of 100 dice than in a throw of 10.)

Mathematically, that can be expressed as

$$\frac{\Delta N(x)}{\Delta x} = -k * N(x) \quad \text{Equation 5}$$

In words, the change in the total number of dice in the x^{th} throw $[\Delta N(x)]$ per throw $[\Delta x]$ equals the number available for the x^{th} throw $[N(x)]$ times the probability of losing each one $[k]$. (The minus sign shows that the number of dice will decrease.)

The solution to Equation 5, which gives $N(x)$ – the number remaining after x throws – is

$$N(x) = N(0) * e^{-k*x} \quad \text{Equation 6}$$

where e (=2.71828..) is the base of the natural logarithms. (No big deal. It's just a number that works out. What's important is that x ends up in the exponent. Aha! Now you see why you used an exponential trend line to fit your data.)

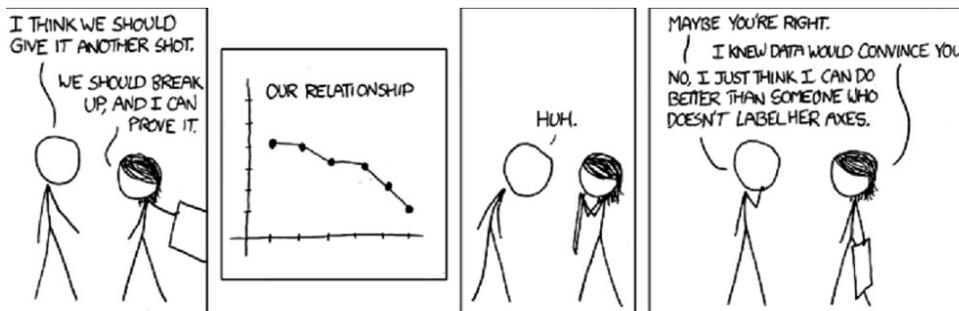
In your experiment, knowing how many numbers were removed on each roll, figure out what was k , the probability of losing each die on each roll? Compare that and the starting number $[N(0)]$ to the value on your trend line.

Each of you should hand in:

- This handout with your name on it.
- Your Excel spreadsheet for each data set. The sheets should contain your data and graph(s). Each graph should have its appropriate trend line and equation displayed.
- Answers to questions asked throughout the hands-on experience such as comparisons between predicted values and your results. You may either type this using Word or handwrite it **neatly**.
- A one or two-paragraph summary of your observations and findings in the lab. Tell me what you learned in the experience and anything you found important. This may also be done in Word or neatly handwritten.

Pre-lab Questions:

1. Neatly (use a ruler to draw the axes) sketch the expected trend line for Equation 3.
2. Re-write Equation 3 for six-sided dice.
3. Repeat Question 1 for six sided dice. Which trend line should decrease faster? Why?



(Don't let this happen to YOU!)