Due: Friday 10/31, 2:10pm, PHYS360 Assignment 9

Reading:
Griffiths, Ch. 4, pages 131-160. Prepare for reading quiz on 10/31 pages 131-160, and especially:
1. Know how to use the equations that generate the associated Legendre functions, Legendre polynomials, spherical Bessel and Neumann functions, the associated Laguerre polynomials and the qth Laguerre polynomial.
2. Given a visual representation of the above functions, be able to evaluate their (complex) values for given \( r, \theta, \phi \). For example, given a plot like in Table 4.2(b), pg 138, be able to evaluate \( P_l^m \).
3. Be able to define:
   - spherical harmonics
   - azimuthal quantum number
   - magnetic quantum number
   - principal quantum number
   - Bohr formula
   - Bohr radius
   - ground state
   - binding energy
   - Lyman, Balmer, and Paschen series

1 A particle is trapped in a cylindrical well, for which the potential is

\[
V = \begin{cases} 
0, & \text{for } 0 < r < a, 0 < z < h \\ 
\infty, & \text{otherwise} 
\end{cases}
\]

Find the allowed energy levels. Hint: Recall that in cylindrical coordinates, the Laplacian is written as

\[
\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2}
\]

Separate the variables, and set the “\( \Phi \)” equation to a constant \( m_\phi^2 \) and the “\( Z \)” equation to “\( k^2 \)”. Rewrite the radial equation in the form

\[
\frac{d^2 R}{d \rho^2} + \frac{1}{\rho} \frac{d R}{d \rho} + \left( 1 - \frac{m_\phi^2}{\rho^2} \right) R = 0
\]

where \( \rho = \lambda r, \lambda^2 = k^2 - k_z^2 \), and \( k^2 = \frac{2mE}{\hbar^2} \).

2 In the ground state of hydrogen, what is the probability that the electron is found inside the Bohr radius?

3 Suppose that \( n = 2 \) and \( l = 0 \). Find the average radius of the electron’s orbit, \( \langle r \rangle \).

4 The state of a hydrogen atom is

\[
\phi = \frac{1}{\sqrt{2}} \psi_{1s} + A \psi_{2p} + \frac{1}{\sqrt{8}} \psi_{3s}
\]

Find \( A \). What is the average energy of the state?
Consider a hydrogen atom in the state with the quantum number \( n \) and \( l \). Show that the dispersion of the distance of the electron from the nucleus is given by

\[
\sqrt{n^2(n^2 + 2) - l^2(l + 1)^2} \frac{2}{2}
\]

Note that the dispersion is defined by \( \sqrt{\langle r^2 \rangle - \langle r \rangle^2} \).

Problem 4.15, part a) only (pg. 156)

PHYSLET: Play with the simulations in section 13.9 and answer the questions (a) through (e) on page 169 (Section 13.9) and Problem 13.4 on page 171. No calculations required.

from Lecture: Show that, for a spherical infinite well, Neumann functions (also solutions to the radial equation) blow up at the origin.