**Due**:Friday 10/31, 2:10pm, PHYS360 Assignment 9

**Reading**:
Griffiths, Ch. 4, pages 131-160. Prepare for reading quiz on 10/31 pages 131-160, and especially:
1. Know how to use the equations that generate the associated Legendre functions, Legendre polynomials, spherical Bessel and Neumann functions, the associated Laguerre polynomials and the $q$th Laguerre polynomial.
2. Given a table of functions above, be able to construct the hydrogen wavefunctions.
3. Given a visual representation of the above functions, be able to evaluate their (complex) values for given $r$,$\theta$,$\phi$. For example, given a plot like in Table 4.2(b), pg 138, be able to evaluate $P_l^m$.
4. Be able to define:
spherical harmonics
azimuthal quantum number
magnetic quantum number
principal quantum number
Bohr formula
Bohr radius
ground state
binding energy
Lyman, Balmer, and Paschen series

**1** A particle is trapped in a cylindrical well, for which the potential is

$$V = \begin{cases} 0, & \text{for } 0 < r < a,\ 0 < z < h \\ \infty, & \text{otherwise} \end{cases}$$

Find the allowed energy levels. Hint: Recall that in cylindrical coordinates, the Laplacian is written as

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

Separate the variables, and set the "$\Phi$" equation to a constant $m_\phi^2$ and the "$Z$" equation to "$k_z^2$". Rewrite the radial equation in the form

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left(1 - \frac{m_\phi^2}{\rho^2}\right)R = 0$$

where $\rho = \lambda r$,$\lambda^2 = k^2 - k_z^2$, and $k^2 = \frac{2mE}{\hbar^2}$

**2** In the ground state of hydrogen, what is the probability that the electron is found inside the Bohr radius?

**3** Suppose that $n = 2$ and $l = 0$. Find the average radius of the electron's orbit, $\langle r \rangle$.

**4** The state of a hydrogen atom is

$$\phi = \frac{1}{\sqrt{2}}\psi_{1s} + A\psi_{2p} + \frac{1}{\sqrt{8}}\psi_{3s}$$

Find $A$. What is the average energy of the state?

**5** Consider a hydrogen atom in the state with the quantum number $n$ and $l$. Show that the dispersion of the distance of the electron from the nucleus is given by

$$\frac{\sqrt{n^2(n^2+2) - l^2(l+1)^2}}{2}$$

Note that the dispersion is defined by $\sqrt{\langle r^2 \rangle - \langle r \rangle^2}$

**6** Problem 4.15, part a) only (pg. 156)

**P** PHYSLET: Play with the simulations in section 13.9 and answer the questions (a) through (e) on page 169 (Section 13.9) and Problem 13.4 on page 171. No calculations required.

**Q** from Lecture: Show that, for a spherical infinite well, Neumann functions (also solutions to the radial equation) blow up at the origin.