Due: Friday, 10/17, 2:10pm, PHYS360 Assignment 7
Note: this assignment is reduced in length

Reading:

1. Griffiths, Ch.3, pg. 114-124. Prepare for Reading Quiz: Friday 10/17, questions on anything in Griffiths, pages 114- 124, but especially:
   - a) understand Dirac notation, 118-120
   - b) be able to represent operators by the matrix elements, pg 120
   - c) understand Example 3.8, pg 120 on the two-state system
   - d) be able to define:
     - bra
     - ket
     - dual space
     - projection operator
   - e) understand the "tidiest" way to express completeness, pg 123

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Problems:

1. Let $|\Psi_1\rangle$ and $|\Psi_2\rangle$ be two orthogonal normalized states of a physical system:

$\langle\Psi_1|\Psi_2\rangle = 0$ and $\langle\Psi_1|\Psi_1\rangle = \langle\Psi_2|\Psi_2\rangle = 1$

and let *A* be an observable of the system. Consider a nondegenerate eigenvalue of *A* denoted by α_n to which the normalized state $|\Phi_\mathrm{n}\rangle$ corresponds. We define $P_1(\alpha_n) = |\langle\Phi_\mathrm{n}|\Psi_1\rangle|^2$ and $P_2(\alpha_n) = |\langle\Phi_\mathrm{n}|\Psi_2\rangle|^2$.

a) What is the interpretation of $P_1(\alpha_n)$ and $P_2(\alpha_n)$?
b) A given particle is in the state $3|\Psi_1\rangle - 4i|\Psi_2\rangle$. What is the probability of getting α_n when *A* is measured?

2. Consider a two-dimensional physical system. The kets $|\Psi_1\rangle$ and $|\Psi_2\rangle$ form an orhonormal basis of the state space. We define a new basis $|\Phi_1\rangle$ and $|\Phi_2\rangle$ by

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle + |\Psi_2\rangle) \qquad |\Phi_2\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle - |\Psi_2\rangle)$$

An operator *P* is represented in the $|\Psi_\mathrm{i}\rangle$-basis by the matrix

$$(a_{ij}) = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}$$

Find the representation of *P* in the basis $|\Phi_\mathrm{i}\rangle$; i.e, find the matrix $\tilde{a}_{ij} = \langle\Phi_i|P|\Phi_\mathrm{j}\rangle$

3. Consider a physical system with a three-dimensional state space. An orthonormal basis of the state space is chosen; in this basis, the Hamiltonian is represented by the matrix

$$H = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

a) What are the possible results when the energy of the system is measured?

b) A particle is in the state $|\Psi\rangle$, represented in this basis as $\frac{1}{\sqrt{3}}\begin{pmatrix} i \\ -i \\ i \end{pmatrix}$. Find $\langle H\rangle$, $\langle H^2\rangle$, and ΔH.

c) Suppose that the energy of the system was measured and a value of $E = 1$ was found. Subsequently we perform a measurement of a variable *A* described in the same basis by

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{pmatrix}$$

Find the possible results of *A*.

d) What are the probabilities of obtaining each of the results found in c)?

4. A particle is in the state $|\psi\rangle$ and its wavefunction is $\psi(x) = \langle x|\psi\rangle$. .

a) Find the mean value of the operator $\hat{A} = |x\rangle\langle x|$.

b) Calculate $\langle x|\hat{p}|\psi\rangle$.

c) Find the mean value of the operator $k_x = [|x\rangle\langle x|\hat{p} + \hat{p}|x\rangle\langle x|]/2m$, where $\hat{p}$ is the momentum operator and *m* is the mass of the particle.